SHORT COMMUNICATION

Dynamics of COVID-19 Transmission: Compartmental-based Mathematical Modeling

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ABSTRACT

The current pandemic of coronavirus disease 2019 (COVID-19) caused by severe acute respiratory syndrome coronavirus 2 (SARS-Cov2) demands scientists all over the world to make their possible contributions in whatever way possible to control this disease. In such health emergency, mathematical epidemiologists are playing a pivotal role by constructing different mathematical and statistical models for predicting different future scenario and their impact on different intervention strategies to policy makers and health legislators. Compartmental-based models (CBM), are a type of transmission dynamic framework, which are one of the most studied models during this pandemic. This communication highlights the role CBM models play for the understanding of COVID-19 transmission dynamics.

Key Words: *COVID-19, Compartmental-Based Models, Emerging Infectious Disease, Transmission Dynamics.*

Mathematical models are known to address specific questions on a disease. Epidemiological models are useful for the prediction of the spread or incidence of an infectious disease. Furthermore, they can also be used to determine the sensitivity of predictions and uncertainties associated with them in parameter values.[1,2,3,4]

The history of modeling epidemics and pandemics starts from 1760 when Daniel Bernoulli, a Swiss mathematician and physicist, formulated a model for the smallpox epidemic. Ronald Ross, in 1906, proposed for the first time an equation-based model to study the transmission of malaria. Extending Ross's work, the concept of compartmental-based modeling was developed by Kermack and Mckendrik in 1927.[4,5] Till now many generalized and extended versions of this useful but basic model for different infectious diseases have been studied.

Encouraged by the effectiveness of disease models, quite a few mathematical models (both equation-based and stochastic), have been constructed for the ongoing corona virus disease 2019 (COVID-19)

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pandemic.[6-16] The predictions are made by considering the impact of distinct interventions that vary from country to country. The forecasts about the disease burden, infectious rate and deaths predicted from these models are not only beneficial for the health care department to better combat the disease but also for the policy makers to plan for the execution of best coping strategies.

Compartmental-based models (CBM)

These are transmission dynamic models, sometimes referred to as population-based models. These types are quite popular among scientists for the modeling of infectious diseases. It is notable that the transmissible nature of a communicable disease is a key trait that differentiates it from non-communicable diseases. The interaction between the susceptible and infectious individuals yields transmission dynamics that require particular considerations when an intervention having influence on the spread of the pathogen is modeled.[3] These type of models are used to study behavioral responses of population to different interventions. They can be deterministic or stochastic in nature.[2,3]

The construction of such models requires two important factors including health states, which represent the pattern of the prevalent disease; and the transition rules in the model which define how the hosts transit between the health states. Additional complexities based, on age, gender or behavior of the host, can also be included in the

model. There are three simple and basic frameworks[1,2]:

Susceptible-Infectious-Susceptible model (SIS), it is useful when recovery from infectiousness is not associated with protection from reinfection and hence, the recovered people are again susceptible to the disease.
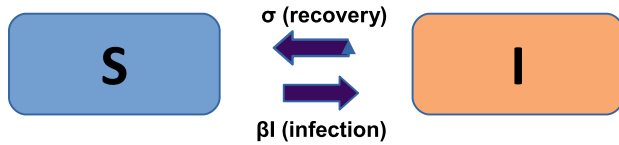
σ (recovery)

S **I**

βI (infection)

Fig 1: Flow chart of SIS compartment model with transmission and recovery parameters

The compartments S and I in Figure 1 represent the health states (i.e., susceptible and infected) of people in population and the arrows denote the transition between the states with the transmission parameter β and recovery rate σ. The rate at which the number of people are decreasing/increasing in each compartment with respect to time is given by the differential equations (1):

$$\frac{dS}{dt} = -\beta IS + \sigma I$$
$$\frac{dI}{dt} = \beta IS - \sigma I$$

(1)

Susceptible-Infectious-Recovered (SIR) model, this type of framework is used when hosts become infectious immediately after infection and possess at least partial immunity to reinfection on recovery. Therefore, an additional compartment (Figure 2) is added in the model which represents the recovered people in population.

βI (infection) **σ (recovery)**

S **I** **R**

Fig 2: Flow chart of SIR compartment model with transmission and recovery parameters

Differential equations for this type with parameters β and σ are given by (2):

$$\frac{dS}{dt} = -\beta IS$$
$$\frac{dI}{dt} = \beta IS - \sigma I$$
$$\frac{dR}{dt} = \sigma I$$

(2)

Susceptible-Exposed-Infectious-Recovered, this framework is suitable for modeling a disease with a period of latency between time of infection and time that an infected individual becomes infectious to others. This model consists of four compartments (Figure 3), representing the susceptible, exposed, infected and recovered individuals.
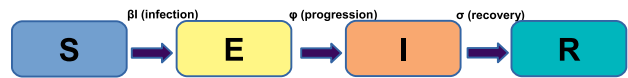
βI (infection) **φ (progression)** **σ (recovery)**

S **E** **I** **R**

Fig 3: Flow chart of SEIR compartment model with transmission, progression and recovery parameters

Due to an additional compartment another variable and parameter φ, denoting the progression rate, is added. Consequently, there is an additional differential equation in this model. The system of equations are given as (3):

$$\frac{dS}{dt} = -\beta IS$$
$$\frac{dE}{dt} = \beta IS - \varphi E$$
$$\frac{dI}{dt} = \varphi E - \sigma I$$
$$\frac{dR}{dt} = \sigma I$$

(3)

The 'recovered' state is sometimes considered as 'removed state' where an infected individual is removed from the Infectious compartment either being recovered and getting immunized from the disease or by death.[2]

After the construction of models, they are often calibrated to see if it fits the observed data. These model parameters are built upon, notified or observed data and sensitivity and uncertainty analyses of the model.[2]

One of the fundamental metrics in the infectious disease epidemiology is the basic reproduction number (often denoted by R_0). It gives the measure of the disease ability to spread in a population. Its threshold value is 1, and if the value is below 1, it is interpreted that no further strict control measure is required while a value above 1 shows that the epidemic will spread. For the basic frameworks, it is calculated as the ratio of the transmission parameter and recovery rate.[3,4]

CBM models and COVId-19

The pandemic situation usually makes it difficult to

study about the disease and make inferences. In such situations, modeling of the disease with the help of some initially available data may provide some insights about the behavior of the disease. Various CBM models have been constructed to study the COVID-19 transmission dynamics.

The most commonly and efficiently used model is the SEIR compartment model (Figure Fig 4) with infection contact rate approximately 0.5 per day and infectious period on average seven days. The latent period varies from region to region from two to five days. Different studies estimated and used different values of the basic reproduction number R_0 based on the scenarios of their countries. The highest median value reported is 5.7 in China.[17] It is obvious that the value of R_0 is not fixed, it changes with the introduction of different interventions, development of immunity and the change in other environmental factors.
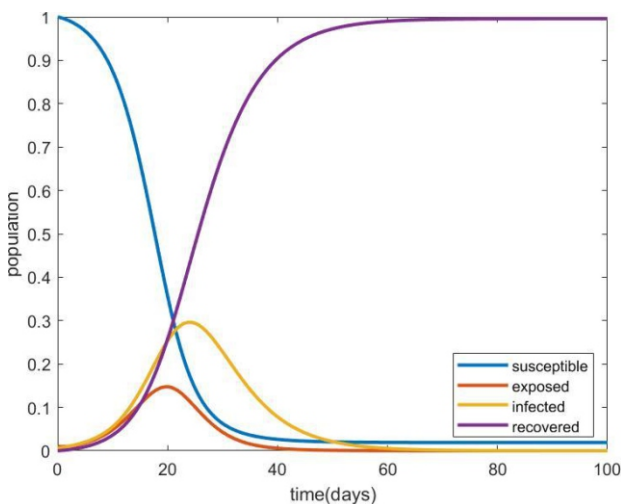

Fig 4: The behavior of population in different compartments during covid-19 outbreak using SEIR model with reproduction number 4.26 (max), latent period 2.5 days and infectious period 7 days.

These studies of compartmental modeling of COVID-19 can be majorly divided into three categories:

1. models constructed and analyzed using different techniques
2. models constructed for studying the impact of different strategies
3. Age-stratified models constructed for studies on the basis of the individuals' age

Predictive Models
Different studies have been done using basic

framework models and their variations.[7,12,13,14,16,18,19,20,21] Some studies have considered an SEIR model as most appropriate for understanding this pandemic as there is a specific time period for the onset of the infectiousness in the host after being infected, while others suggested a simple SIR model (having no latent stage). Extended versions of basic SEIR model have been developed by including symptomatic and asymptomatic compartments, or further by adding hospitalized, critical and dead individuals.[22,23,24] Most of these models are constructed for the estimation of the reproduction number, short term predictions of the disease burden and/or analyzing the sensitivity of the model with respect to variable parameters. These studies suggest that the contact pattern among the individuals plays a vital role in this disease spread. In addition, the contact rate of the susceptible and infectious is the most critical factor for mitigating the intensity of COVID-19.

Models with interventions impact
Many models have studied the effect of non-pharmaceutical interventions on the COVID-19 using basic as well as generalized models in order to reduce the basic reproduction number and to reduce the severity of the disease.[15,25,26,27,28,29,30] These include face masking, social distancing, different locked down strategies, quarantine of exposed cases, isolation of infected cases and isolation of asymptomatic people. Other factors like rapid testing, trace of contacts (asymptomatic or unreported cases), environmental factors like seasonality factors and transmission through surfaces and limited medical resources in low income countries were also included in the models.[15,31] Moreover, the effectiveness of the state control measures has been evaluated through these models.[32] In addition to other interventions, adoption of face masks has been proved to be beneficial as their inclusion along with other measures decrease the transmission rate of the disease.[26] The absence of home quarantine strategy results in an increase of infected people, thus affecting adversely the health systems of the countries.[27] While these findings show us that these interventions in one way or another are helpful for slowing down the spread of disease, the lifting of these strategies can result in the rapid rebound of the pathogen.[10,33] We have already experienced that

in Pakistan, the lifting of locked down after Ramadan resulted in a sudden increase in the cases of COVID-19 patients. Hence, the timings when these strategies are implemented during the pandemic and for how long we retain these strategies are also very important.[8] These measures are taken at the cost of societal disruptions and mental health of the individuals. It may not be possible to minimize the severity of the disease and at the same time reduce the economic and societal disruptions. As suggested by one of the study in US[32], a stepping down strategy would be a feasible idea to reduce the social distancing (SD) over a period of two years.

Age-stratified Models

A noticeable attribute that we come across during the analyses of COVID-19 is that less number of children were affected by COVID-19,[8,34] but whether they are less infectious or less susceptible, or both, is still an ongoing debate. Most of the studies suggest that adults are the main source of transmission of the virus but the true scenario of age-specific transmissibility of corona virus is not yet known.[35] According to a study based on the data from six different countries, China, Italy, Japan, Singapore, Canada, South Korea and a generalized SEIR model, the children below 20 years of age were found to be half susceptible than adults, and even if they get infected are more prone to be asymptomatic.[36]

Limitation of CBM models

CBM modeling is a useful mechanism for understanding the dynamics of an epidemic, but it should be noted that there is no 'all in one' framework which can address all questions. There are advantages of using these models for COVID-19, but like any other mathematical model, they have limitations. These are subject to some uncertainty since they are hypothesized and the values of the parameters can only be approximated. Such models are dependent on the data that has been reported from the different institutes. Hence, the reliability and authenticity of the data is very essential. However, during an ongoing pandemic, it is difficult to get precise data, from underdeveloped regions. The lack of precision in the notified data further leads to imprecision in the estimated parameters. However, for some parameters it is possible to get a range in which the parameters lie.

It is recommended to model the disease with additional variables for a clear understanding of the disease. A delineated scenario of the current situation requires more compartments which increases the number of differential equations. In case of a short-term projection, this does not create a problem but a long term projection will require sophisticated simulations.

Another limitation is that any further generated data during this pandemic could change the parameter estimates and that could lead to different conclusions.

Conclusion

SARS-Cov2 is a communicable disease that has spread globally and continues to spread via direct and indirect contact between the individuals. In the absence of any proper treatment or vaccination, interventions like social distancing and personal protection measures like masking and physical hygiene are important and should be observed in order to reduce the severity of this disease. The results from different studies indicate that there is an immense impact of these techniques on the transmission dynamics of the disease. Additionally, a complete locked-down situation is not sustainable for underdeveloped countries. An intermittent locked-down strategy or lock down for certain time periods and in certain areas (preferably congested ones) would be a better idea to cope with the situation. For future investigations, it is recommendable to construct a hybrid model for the spread of SARS-Cov2 using both Compartmental-based and Individual-based models, in order to get the benefits of both types of modeling. Furthermore, modeling with the impact of vaccination could also be explored.

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